

The Never-Ending War

Marshall King, Sinjini Sengupta, John Weeks, and Phil Yasskin

Texas A&M University

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Texas A&M Math Circle

Founded in 2011 by PY, Frank Sottile, and Alex Sprinston

55 students signed up with 30 showing up each Saturday (numbers today are significantly lower than pre-COVID years)

1-3pm Discovery Learning activities, 3-4pm Problem Solving (AMC/MATHCOUNTS/etc competition prep)



Rules of Sticks

You and your opponent both start with both hands held out and one finger held out on each hand

Players take turns tapping one of their hands against one of the opponent's hands

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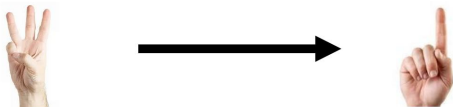
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The goal is normally to be the last player with a hand remaining in play, but we want to find out if the game can go on forever

A Quick Example



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Try it Yourself!

Now you try the game with a partner using 2 hands each. Start in this configuration:

Player 1



Player 2



Can you make the game last forever? If so, how do you know it will never end?

Learning From Our Students

The image shows two rows of handwritten mathematical work on a piece of paper. Each row consists of several equations separated by vertical bars, resembling a sequence of steps or a derivation.

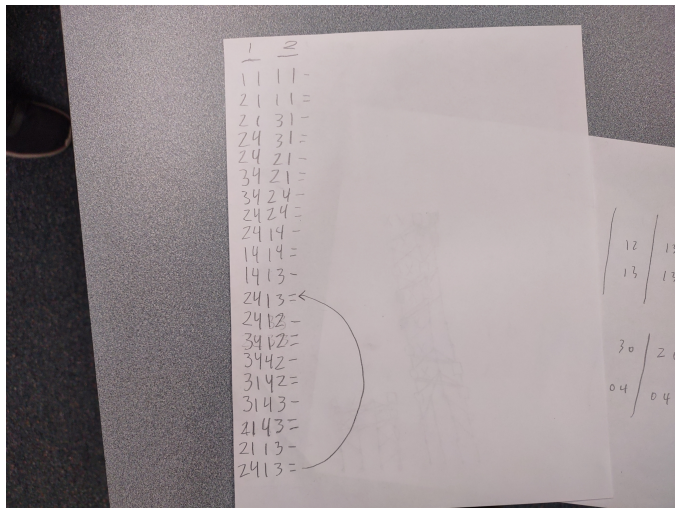
The first row contains the following equations:

$$\begin{array}{l} 1 \quad 1 \quad | \quad 1 \quad 2 \quad | \quad 1 \quad 2 \quad | \quad 1 \quad 3 \quad | \quad 1 \quad 3 \quad | \quad 1 \quad 3 \quad | \quad 1 \quad 0 \\ 1 \quad 1 \quad | \quad 1 \quad 1 \quad | \quad 1 \quad 3 \quad | \quad 1 \quad 3 \quad | \quad 2 \quad 3 \quad | \quad 2 \quad 3 \end{array}$$

The second row contains the following equations:

$$\begin{array}{l} 1 \quad 0 \quad | \quad 3 \quad 0 \quad | \quad 3 \quad 0 \quad | \quad 2 \quad 0 \quad | \quad 2 \quad 0 \quad | \quad 3 \quad 0 \quad | \quad 3 \quad 0 \\ 2 \quad 4 \quad | \quad 2 \quad 4 \quad | \quad 0 \quad 4 \quad | \quad 0 \quad 4 \quad | \quad 0 \quad 1 \quad | \quad 0 \quad 1 \quad | \quad 0 \quad 4 \end{array}$$

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Rules of Beggar-My-Neighbor

Start with a deck of standard playing cards. Shuffle and distribute them evenly between two players.

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The first player plays the top card from their deck face-up in the center of the table. If it is a “penalty card,” the opponent must “pay the penalty” by playing as many cards as the “penalty value” from the top of their deck. If it is not a penalty card, the second player repeats this step with the top card from their deck.

The penalty cards and their values are:

A = 4 cards

K = 3 cards

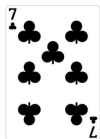
Q = 2 cards

J = 1 card

Rules of Beggar-My-Neighbor

If a player plays a penalty card while paying the penalty, they stop playing cards and their opponent must start paying the new penalty based on the penalty card just played.

If a penalty is fully paid without playing a penalty card in the process, the player who last played a penalty card wins the “trick.” They receive all cards in the center of the table and place them on the bottom of their deck without changing their order, then begin the next trick by playing the top card of their deck into the center.



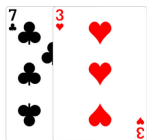
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Red Cards: Player 2

The game ends when one player runs out of cards.

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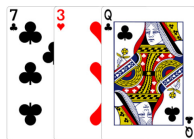
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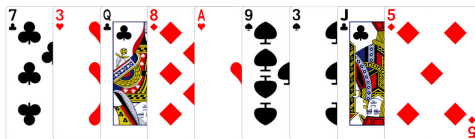
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Black Cards: Player 1

Red Cards: Player 2

Player 1 Wins the Trick

The game ends when one player runs out of cards.

A Simple Example Game

You and your partner should have a (small) deck of cards.

Remove the deck from the box without shuffling and hold it face-down.

There should be a slip of paper in the middle of the deck. Give the cards above this slip to player 1 and the cards below to player 2.

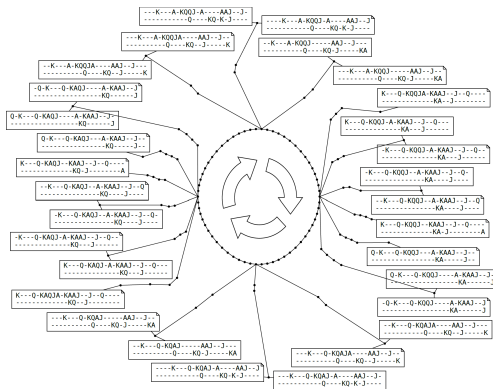
Start playing the game, with player 1 going first. Cards should be played from the top of the decks when they are face-down.

What do you notice?

Could We Use a Full Deck?

Is it possible to construct a never-ending game using the full 52-card deck where the game begins with cards split evenly between the two players?

(Casella et al., 2024) - Yes! In fact, there is a whole family of 30 of these games which all lead into the same cycle of 62 tricks:



Math Circle Links

Our Website

(<http://mathcircle.tamu.edu/>)



MYMathApps

(tinyurl.com/MYMathApps)



Math Circle Activity
Spreadsheet

(tinyurl.com/MathCircleSpreadsheet)



B. Casellas, P. Anderson, M. Kleber, R. Mann, R. Nessler, W. Rucklidge, S. Williams, and N. Wu. *A Non-Terminating Game of Beggar-My-Neighbor*, preprint (2024), arXiv:2403.13855.

Thank you!